Local reasoning in fuzzy attribute graphs for optimizing sequential segmentation

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Abstract. Spatial relations play a crucial role in model-based image recognition and interpretation due to their stability compared to many other image appearance characteristics. Graphs are well adapted to represent such information. Sequential methods for knowledge-based recognition of structures require to define in which order the structures have to be recognized. We propose to address this problem of order definition by developing algorithms that automatically deduce sequential segmentation paths from fuzzy spatial attribute graphs. As an illustration, these algorithms are applied on brain image understanding.

1 Introduction

Knowledge on the spatial organization of a scene carries important information for analyzing and interpreting images of this scene. Spatial relations play a crucial role in this context, since they are less prone to variability than object appearance or shape. Using this knowledge, often represented in symbolic forms, in high reasoning processes requires to link semantic knowledge with low level information extracted from images. Graph representations are well adapted to solve this semantic gap problem.

In [1], spatial relations and graph-based representations have been used for recognizing structures in a progressive way: the recognition of a structure is driven by its relations to previously recognized structures; these relations are encoded in a graph representing generic knowledge. This allows recognizing "difficult" structures at later stages, once more information has been accumulated. In this work, the order in which structures are recognized is defined in a supervised way. Figure 1 shows some segmentation results obtained with a manually defined order.

In this paper, we propose to automate this step, and to infer automatically segmentation paths using reasoning algorithms in the graph. The idea is to start

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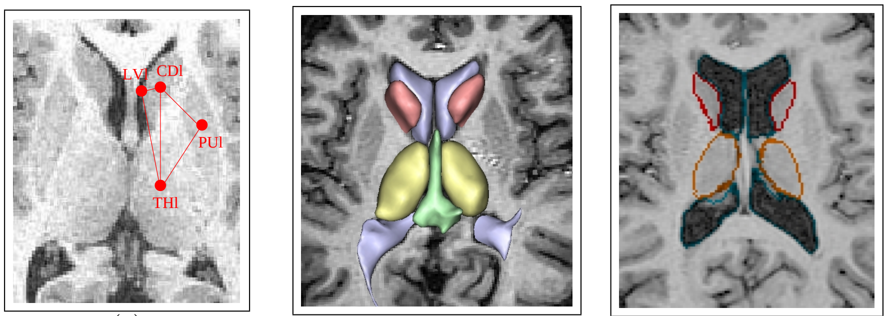


(a) (b) (c)

Fig. 1. (a) Slice of a 3D brain magnetic resonance image (MRI). Marked structures are: LVl lateral ventricle, CDl caudate nucleus, THl Thalamus and PUl Putamen. (b, c) Segmentation results from [1].

from a structure, represented as a node in the graph, which is known for being easy to segment and recognize in the images, and to automatically deduce an ordered sequence of structures to be recognized.

A typical application is brain image interpretation, where the domain knowledge involves intensively spatial relations, as acknowledged by neuro-anatomy textbooks [2]. These relations are relatively stable, and exhibit less inter-individual variability than characteristics of the anatomical structures. Graph representations have been used in particular to drive specific recognition procedures (see e.g. [1, 3, 4] among others). However, in pathological cases, generic knowledge is not always valid and information about the pathology has to be used in order to adapt the reasoning process.

The structure of this paper is as follows. We first describe in Section 2 the graph model, specifically for representing anatomical brain knowledge, along with the fuzzy attributes of edges representing spatial relations. Our contribution on graph-based reasoning is presented in Section 3 for the healthy case. Preliminary results are discussed in Section 4. Some hints towards adaptation of the proposed approach to pathological cases are provided in Section 5.

2 Graph model

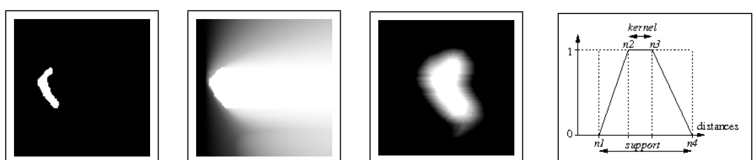
In this paper, we follow the same approach as in [1], and we propose an original method to determine automatically the order in which structures should be segmented, using the spatial relations represented as edge attributes of a graph (nodes represent individual objects, such as anatomical structures in the brain example). Note that this way of using the graph is very different from classical graph matching approaches, widely developed for structural recognition. Let us now summarize the adopted formalism for representing spatial relations.

Fuzzy representations are appropriate to model the intrinsic imprecision of several relations (such as “close to”, “behind”, etc.), the potential variability (even if it is reduced in normal cases) and the necessary flexibility for spatial reasoning [5]. Two kinds of questions are raised when coping with spatial relations: (i) given two objects (possibly fuzzy), determine the degree of satisfaction of a relation; (ii) given one reference object, define the region of space in which a relation to this reference is satisfied (to some degree). In this paper, we deal mainly with the second question.

Therefore we rely on spatial representations of the spatial relations: a fuzzy set in the spatial domain $\mathcal{S}$ defines a region in which a relation to a given object is satisfied. The membership degree of each point to this fuzzy set corresponds to the satisfaction degree of the relation at this point [5]. Figure 2 depicts an example.

We now describe the modeling of the main relations that we use: adjacency, distances and directional relative positions.

A **distance** relation can be defined as a fuzzy interval f of trapezoidal shape on $\mathbb{R}^+$, as illustrated in Figure 2. A fuzzy subset μ_d of the image space $\mathcal{S}$ can then be derived by combining f with a distance map d_A to the reference object A: $\forall x \in \mathcal{S},\ \mu_d(x) = f(d_A(x))$, where $d_A(x) = \inf_{y \in A} d(x, y)$.



a) b) c) d)

Fig. 2. (a) 2D view of a 3D binary lateral ventricle. (b) Fuzzy spatial representation of "Right of the lateral ventricle". (c) Fuzzy subset corresponding to "Near the lateral ventricle". (d) Trapezoidal fuzzy interval.

Directional relations are represented using the "fuzzy landscape approach" [6]. A morphological dilation δ_{ν_α} by a fuzzy structuring element ν_α representing the semantics of the relation "in direction α" is applied to the reference object A: $\mu_\alpha = \delta_{\nu_\alpha}(A)$, where ν_α is defined, for x in $\mathcal{S}$ given in polar coordinates (ρ, θ), as: $\nu_\alpha(x) = g(|\theta - \alpha|)$, where g is a decreasing function from $[0, \pi]$ to $[0, 1]$, and $|\theta - \alpha|$ is defined modulo π. This definition extends to 3D by using two angles to define a direction. The example in Figure 2 has been obtained using this definition.

Adjacency is a relation that is highly sensitive to the segmentation of the objects and whether it is satisfied or not may depend on one point only. Therefore we choose a more flexible definition of adjacency, interpreted as "very close to". It can then be defined as a function of the distance between two sets, leading to a degree of adjacency instead of a Boolean value: $\mu_{adj}(A, B) = h(d(A, B))$ where $d(A, B)$ denotes the minimal distance between points of A and B: $d(A, B) =$

$\inf_{x \in A, y \in B} d(x, y)$, and h is a decreasing function of d, from $\mathbb{R}^+$ into $[0, 1]$. We assume that $A \cap B = \emptyset$.

3 Graph-based reasoning in normal cases

In this section, we deal with normal cases. The aim of the reasoning in the graph is to select the "best" path between a reference structure and a target structure to be segmented and recognized in an image, by exploiting the information encoded in the graph. Note that the number of simple paths (without loops) between two structures is finite. Path extraction is known as an intractable task but we limit our experiments to small graphs. Extensions to larger graphs require to address this issue.

The reference structure, in the case of MRI images of the brain, can typically be the lateral ventricles, which are easy to segment in such images. The notion of "best" path refers to the constraints of the segmentation process: it should allow segmenting a structure in the path based on relations to previous structures in the path, as done in [1] (based on manually defined paths). We propose two methods:

- the first one is based on the evaluation of the relevance of each spatial relation between two structures independently, and on the optimization of the path according to a criterion involving this relevance measure;
- in the second one, we estimate each path globally and select the best one according to another criterion.

3.1 Evaluating edge relevance

In this part, we present a criterion of relevance as well as two different methods for path selection.

In the following, $G = (V, E)$ is an attributed relational graph, with V the set of nodes and E the set of edges. An edge interpretor associates to each edge e a fuzzy set μ_{Rel}, defined in the spatial domain, representing the spatial relation carried by this edge to a reference structure as defined in [6]. Similarly a fuzzy set μ_{Obj} is attached to each node.

Relevance criterion The relevance of a spatial relation should represent the adequation between μ_{Rel} and μ_{Obj}, i.e. the degree to which the target object fits in the region where the relation to the reference object is satisfied. The comparison measures and their classification according to [7] provide an appropriate formal framework for this purpose.

For both the reference structure, used to compute μ_{Rel}, and the target object, used for μ_{Obj}, we need an a priori knowledge from an anatomical atlas or from a set of pre-segmented images.

M-measure of satisfiability: We use a M-measure of satisfiability [7] defined as:

$$f(Rel, Obj) = \frac{\sum_{x\in\mathcal{S}} \min(\mu_{Rel}(x), \mu_{Obj}(x))}{\sum_{x\in\mathcal{S}} \mu_{Obj}(x)}. \tag{1}$$

where $\mathcal{S}$ denotes the spatial domain. It measures the precision of the position of the object in the region where the relation is satisfied and is maximal if the whole object is included in the kernel of μ_{Rel}. But the size of the region where the relation is satisfied is not restricted and could be the whole image space. Note that if the object is crisp, this measure reduces to $\frac{\sum_{x\in Obj} \mu_{Rel}(x)}{\sum_{x\in\mathcal{S}} \mu_{Obj}(x)}$.

Path selection Once every edge has been valued with the proposed relevance measure, path selection is achieved with classical algorithms, such as shortest path or maximal flow. Nevertheless, these algorithms have to be adapted to our purpose.

Shortest path: The shortest path algorithm leads to a global optimization, but does not account for potential disparities between edges. A globally satisfactory path can include an edge with a low relevance value. Moreover, this algorithm favors paths with a reduced number of nodes, hence leading to less segmented structures. The adaptation we propose consists in normalizing the cost of each path by its length (in terms of number of nodes).

Let $\mathcal{F}$ denote the set of the fuzzy sets over the spatial domain. Let $f : \mathcal{F} \times \mathcal{F} \to \mathbb{R}$ be a real valued cost function, here a satisfiability measure. The shortest path between two nodes v and v' is the path $\hat{p}$ solution of:

$$\min_{p\in\mathcal{P}} \left(\frac{\sum_{e\in p}(1 - f(\mu_{Rel}, \mu_{Obj}))}{card(p)}\right) \tag{2}$$

where e is an edge in the path p, $\mathcal{P}$ is the set of paths from v to v', μ_{Obj} is the target node of edge e, μ_{Rel} is the fuzzy set derived from e and $card(p)$ is the number of edges in p.

Maximal flow: We adapt the classical maximal flow notion [8] in order to take the weakest edges into account without penalizing the most informative paths. This is expressed as the maximization of the minimal value along the path:

$$\max_{p\in\mathcal{P}}(\min_{e\in p}(f(\mu_{Rel}, \mu_{Obj}))) \tag{3}$$

where f is again a satisfiability measure. This formulation allows avoiding paths including relations which are not well satisfied.

3.2 Globally evaluating path relevance

Instead of evaluating the relevance for each edge, we propose in a second method to evaluate the relevance of a whole path by merging spatial knowledge along this path.

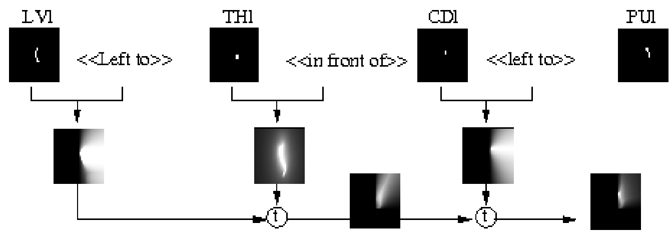


Fig. 3. Merging spatial relations. For each relation carried by an edge on a path, we compute its representation, using a priori knowledge for the structure. Representations of all relations along the path are then merged with a t-norm (here a minimum).

Merging spatial knowledge In this approach, we combine information along the path with prior knowledge derived from an anatomical atlas, as illustrated in Figure 3. For each structure in the atlas and each spatial relation encoded in the graph we compute the fuzzy set representing the region where the relation to this structure is satisfied, as previously. Note that it is relevant to merge different relations (distance and direction for example) since all relations use the same representation framework i.e. fuzzy sets in the spatial domain. The fuzzy sets obtained for all pairs structure/relation along the path p are combined using a t-norm (a conjonctive fusion operator):

$$\mu_p = t[\mu_{Rel_i^p}, i = 1...N^p] \tag{4}$$

where t is a t-norm and p a path composed of N^p relations. In our experiments, we use the minimum t-norm.

Path evaluation using entropy In this approach, the path selection method we propose relies on a a fuzziness measure, in order to choose the "less fuzzy" path. As a fuzziness measure, we choose the fuzzy entropy measure [9]:

$$H(\mu_p) = -K(\sum_{x_i \in \mathcal{S}} \mu_p(x_i) \log \mu_p(x_i) + \sum_{x_i \in \mathcal{S}} (1 - \mu_p(x_i)) \log(1 - \mu_p(x_i))) \tag{5}$$

where μ_p is the fuzzy set resulting from the combination of all relations along p and k is a normalizing constant.

The best path $\hat{p}$ is then the path which achieves the minimum of fuzzy entropy:

$$\hat{p} = arg \min_{p \in \mathcal{P}} (H(\mu_p)). \tag{6}$$

Note that this measure is meaningful for representations of relations that are more fuzzy if they are less focused. It is actually the case with our model of relations. For instance, it would be useless to apply this criterion on large crisp regions which would lead to a zero entropy value even if these regions are very extended and of limited help to constrain the segmentation.

4 Results and discussion

Experiments have been carried out on a small graph presented in Figure 4 containing four cerebral structures: the lateral ventricle (taken as the reference structure), the caudate nucleus, the thalamus and the putamen (the target structure in our experiments). All these structures exist in both brain hemispheres, but only the left side is considered in the reported experiments. Note that the extraction of the structures is supposed to exhibit the same difficulty level.

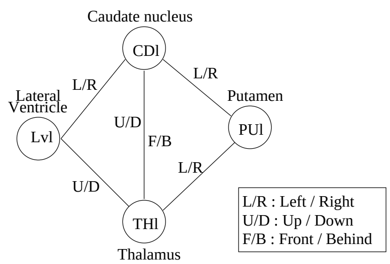
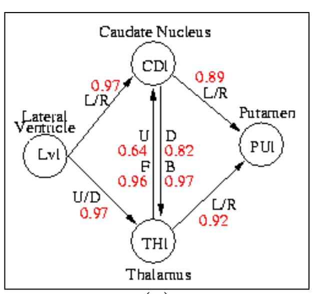


Fig. 4. Small graph used in the experiments

The edges encode only information about directional relative position in these preliminary experiments. Extending our approach to other binary spatial relations can be achieved in a straightforward manner.

4.1 Edge valuation



(a)

edge			Satisf.
Lat. ventricle	“Down of”	thalamus	0.97
Lat. ventricle	“Left of”	cau. nucleus	0.97
Cau. nucleus	“Behind of”	thalamus	0.97
Thalamus	“In front of”	cau. nucleus	0.96
Thalamus	“Left of”	putamen	0.92
Cau. nucleus	“Left of”	putamen	0.89
Cau. nucleus	“Down of”	thalamus	0.82
Thalamus	“Up Of”	cau. nucleus	0.64

(b)

Fig. 5. (a) Edges valuation with a measure of satisfiability. (b) Edge ranking according to this measure.

Measures of satisfiability obtained for each edge are presented in Figure 5. The best path according to the satisfiability criterion with normalized shortest path and flow measure is: LVl “left of” CDl “behind” THl “left of” PUl. This path is exactly the one that was previously defined by hand in [1] and that led to the results shown in Figure 1.

Another path with the highest score is: LVl “down of” THl “left of” PUl. This path is less intuitive since it involves a few number of structures. For practical purposes, if several paths exhibit the same global score, the longest path (in terms of number of nodes) is retained.

4.2 Merging of spatial relations

The best path according to the entropy criterion is: LVl “down of” THl “up of” CDl “left of” PUl. Figure 6 shows a view of the resulting representation of this path. This path contains several changes in direction which explain the strongly focused resulting fuzzy region. More generally, paths with several changes in direction get low entropy while simpler paths get high entropy.

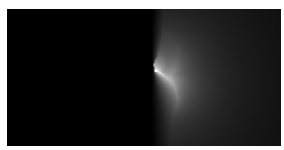
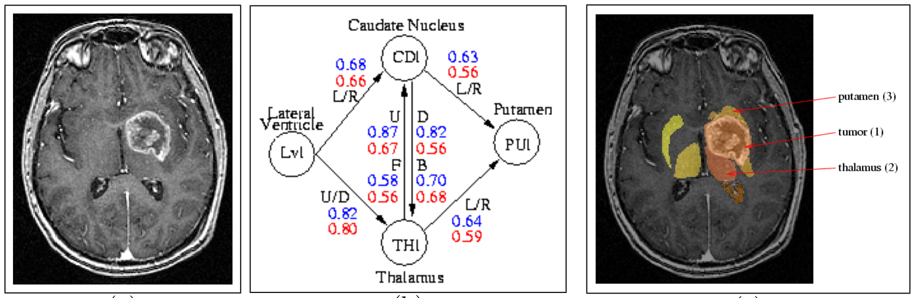


Fig. 6. 2D slice of 3D representation for path LVl “down of” THl “up of” CDl “left of” PUl after merging all spatial relations.

5 Graph-based reasoning in pathological cases

The approaches introduced in Section 3 are not directly applicable in the case of the presence of a pathology and require some adaptation. For instance, the presence of a tumor may induce an important alteration of the appearance and morphometric characteristics of the structure. Although spatial relations are more stable, still modifications of the structural information may occur. Figure 7 presents an example of a pathology in a MRI brain image, illustrating the impact of the tumor on the surrounding structures.

It has been shown in [10] that some spatial relations are more stable than others. A pathology-dependent paradigm has been introduced to adapt a generic reasoning process to specific cases by addressing the fundamental question: given a pathology, which spatial relations do remain stable and to which extent? For this purpose, we designed a computational framework for learning spatial relation stability from a database constituted of healthy and pathological cases, where the main anatomical structures were manually segmented. The degree of stability is inferred from the comparison (using a M-measure of resemblance) between the learned spatial relations for pathological cases and for healthy ones.



(a) (b) (c)

Fig. 7. (a) Axial view of MRI with a tumor close to the lateral ventricle and grey nuclei. (b) Degree of stability learned with a class of similar tumor (in blue). Resulting weighted satisfiability measures (in red). With the sortest path method, the selected path becomes Ventricle "Down Of" thalamus "Left Of" Putamen. (c) Segmentation of the putamen. The tumor is first extract then the thalamus and finally the putamen

In this work we exploit the degree of stability concept to adapt the reasoning approaches designed for healthy cases to pathological ones. This can be achieved in several ways.

The initial graph is filtered so that the spatial relations with a low degree of stability are removed. Then the proposed methods are applied on the filtered graph instead of the initial one. This approach is very severe and does not leave significant place to flexibility, an important property in reasoning and decision making paradigms.

In the second method, the degree of stability is taken into account as an edge attribute and is considered in the cost calculation of the proposed approaches. This approach is a direct extension of the methods proposed for healthy case, and its implementation is straightforward. The integration of the degree of stability must be achieved in a way so that the paths involving pathological or altered structures are penalized. For instance, when using the shortest path method, a weight proportional to the degree of stability is assigned to $f(\mu_{Rel}, \mu_{Obj})$. Figure 7 b) presents the degree of satisfiabity (in blue) learned for each edge in the case of the tumor like the one presented in Figure 7 a) and the weighted measures in red. In this case, the selected path becomes ventricle "Down Of" thalamus "Left Of" Putamen. Figure 7 c) presents a segmentation of the putamen with the same order.

In the global approach, the influence of a relation is decreased by extending its spatial extension (for instance using a fuzzy dilation), so as to increase the resulting degree of fuzziness, and thus unfavoring paths including this relation.

This last idea, of extending the fuzzy representation, is the basis of the third method we propose. Since the fuzzy representation of spatial relations presents the advantage of being flexible in the way they can be constructed, this construction could be correlated to the degree of stability. For instance, the definition of "near the lateral ventricles", as explained in Section 2, is modified by extending the fuzzy interval according to the degree of stability (the less the stability, the more the extension and the more fuzzy). This induces both a lower resemblance and more fuzziness, hence decreasing the relevance of paths including this relation in both approaches.

These approaches are currently being tested on different pathological cases.

6 Conclusion

The main contribution of this paper is to show that the order of structures in a sequential segmentation process can be deduced automatically using graph-based reasoning. We proposed relevance measures of segmentation paths based on fuzzy representations of spatial relations. As an illustration, we applied our method on a small graph representing brain structures. The results are promising since the best path actually allows driving the recognition and segmentation procedure in 3D MRI brain images.

Extensions to the pathological cases are proposed, based on the impact of the pathology on the spatial relations. This part will be further investigated in future work. Applications on larger graphs will also be carried out, which may require to address potential combinatory optimization issues.

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